Landsat 8 / Sentinel 2 Fusion Technics

Algorithm Theoretical Basis Document (ATBD)

Time Series Consolidation and Densification using Dynamic Time Warping Technic

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<tr>
<td>Prepared by:</td>
<td>G. Salgues S. Saunier</td>
<td>Magellium</td>
<td>09/10/2015</td>
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<td>09/10/2015</td>
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<td>S. Saunier</td>
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<td>ESA Contract No VEGA/AG/14/01720</td>
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<td>G. Davis</td>
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<td>J. Swinton</td>
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<td>1</td>
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<td>09/10/2015</td>
<td>PM3 Presentation and Document Review Meeting with ESA.</td>
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1 Purpose

This Algorithmic Theory Baseline Definition is prepared by Magellium for the Project “Landsat 8 / Sentinel 2 Fusion Technics”, which is the subject of the contract “VEGA/AG/14/01720 / CCN01” issued by the European Space Agency further on called ESA.

This technical specification (TS) is organized in six sections and four appendices; in the following a brief description of the content sections is provided, with the relationships between them:

- Chapter 1: Purpose. This is this section.
- Chapter 2: References. This section mentions applicable and reference documents related to this one.
- Chapter 3: Terms and Abbreviations. This section defines terms which are used within this document.
- Chapter 4: Context and Scope of this document. This section explains the context of using fusion technics between data from two different sensors.
- Chapter 5: Time Series Consolidation Methods. This section states the algorithmic theory for consolidation of mono sensor time series.
- Chapter 6: Time Series Densification Methods. This section states the algorithmic theory behind the fusion between two time series originated from two different sensors, so called time series densification.
2 References

2.1 Applicable documents

The following documents are fully applicable together with this document.

<table>
<thead>
<tr>
<th>Id.</th>
<th>Ref.</th>
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*tab 1. Applicable Documents.*

2.2 Reference documents

The following documents, though not a formally part of this document, amplify or classify its contents.

<table>
<thead>
<tr>
<th>Id.</th>
<th>Ref.</th>
<th>Description</th>
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<tbody>
<tr>
<td>[RD-1]</td>
<td>Testing report</td>
<td></td>
</tr>
<tr>
<td>[RD-2]</td>
<td>Dr Eamonn Keogh, Exact Indexing of Dynamic Time Warping, power point presentation.</td>
<td></td>
</tr>
<tr>
<td>[RD-6]</td>
<td>F. Petitjean Dynamic time warping : theoretical contributions for data mining, application to the classification of satellite image time series</td>
<td></td>
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<td>[RD-7]</td>
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<td>Description</td>
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**tab 2. Reference Documents.**
3 Terms, Definition and Abbreviated Term

3.1 Terms and Definitions

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clustering</td>
<td>The clustering is a way to aggregate pixels around a center. It is commonly used in classification. In our cases, this center corresponds to the average time series. The clustering is operated with a classifier, K-MEANS. As a result, the input ROI is now breakdown into several regions, each on belonging to a cluster.</td>
</tr>
<tr>
<td>Data stack</td>
<td>Time series of Image Data observed over the same ROI.</td>
</tr>
<tr>
<td>DBA</td>
<td>DTW Barycentre Averaging (DBA) is a time series averaging technics. Under constraints, DBA iteratively refine an average sequence. In the scope of our work, the DBA is the best method for time series averaging for DTW. The average sequence corresponds to a centroid.</td>
</tr>
<tr>
<td>Dynamic Time Warping</td>
<td>The Dynamic Time warping is a distance measure between two time series. The DTW is used to align sequence on a common time-axis by using dynamic programming. The optimal warping path (between two series) satisfies three basic constraints: boundary condition, continuity and monotonicity ([RD-2]). The optimal warping path is found as satisfying the minimal overall distance. The DTW considers Temporal dimension as part of the distance and is particularly convenient for the analysis of the time series. The DTW is used to compute the average time series, conjointly with DBA. The DTW can be used to identify pair between two time series and will be</td>
</tr>
<tr>
<td>Occluded Area</td>
<td>An occluded area in an image includes pixels for which the values are not nominal with respect to neighbor ones due to cloud, shadow....</td>
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</tbody>
</table>
### Terms, Definition and Abbreviated Term

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>In case of gap filling, a temporal block should be carefully selected</td>
<td>Several temporal patches may have occluded area at the same location, but others ones should have to be free from occlusion. It is mandatory in case reference block database is not available.</td>
</tr>
<tr>
<td>The figure below shows a temporal block (green) with occluded area included.</td>
<td></td>
</tr>
<tr>
<td>The assumption is that within the occluded area there are pixels for which values are very close to neighbors ones (from non-occluded pixels). For this reason the temporal block should be carefully selected.</td>
<td></td>
</tr>
<tr>
<td>Considering a data stack, the average time series is more representative than on time series itself, associated with one pixel.</td>
<td></td>
</tr>
<tr>
<td>Temporal information is used as additional information, and DTW conjointly with DBA is used for clustering. As shown in figure below.</td>
<td></td>
</tr>
<tr>
<td><strong>Reference Block</strong></td>
<td>The baseline idea of reference block is to collect centroid considered as reference for different category of regions over regions. It will be subsequently used as centroid during clustering of an incoming temporal block.</td>
</tr>
<tr>
<td><strong>Temporal Block</strong></td>
<td>A temporal block is a data stack of images of a same region of interest (ROI). ROI is corresponding to a geographic area in an image. The figure below shows a schematic of a temporal block observed at different dates. At t1, the temporal block includes occluded area. The proposed</td>
</tr>
</tbody>
</table>
### Term | Meaning
--- | ---
$t_0$ | A temporal block is a sequence of temporal patches.
$t_1$ | A temporal patch is a part of temporal block. It is the image at a given time. A temporal patch is characterized with the following information:
- Observation date
- Area Extent/footprint
- Sensor metadata
- Image data
$t_2$ | Temporal patch with occluded area included

**Temporal Patch**
- A temporal patch is a part of temporal block. It is the image at a given time.
- A temporal patch is characterized with the following information:
  - Observation date
  - Area Extent/footprint
  - Sensor metadata
  - Image data

**Time series**
- It is a sequence of points that are chronologically ordered. The time series refer to a specific geographical location in an image (pixel) or an ROI (mean value over the ROI).

**“Mono TS” / “Multi TS**
- ‘Mono TS’ stands for Mono sensor Time series.
- ‘Multi TS’ stands for Multi sensor Time series, it is the fuzzed time series.

**Transfer function**
- Herein, the transfer function between the two sensors S1 and S2 is used to map Sensor 1 time series to Sensor 2 time series and then densify time series over a given ROI. It is basically based on:
  - Modelling of local variation of Sensor 1 time series, local because it is related to the period that includes the date for which estimate is needed;
  - Time warping function between Sensor 1 time series and Sensor 2 time series.
- The transfer function may account for spectral convolution mismatch; if the overlapping is weak. The sensitivity with respect to this parameter should be evaluated, notably by using the Spectral Band Adjustment Factor (SBAF).


3.2 Abbreviated Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>AD</td>
<td>Applicable Document</td>
</tr>
<tr>
<td>ATBD</td>
<td>Algorithm Technical Baseline Document</td>
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<tr>
<td>E2E</td>
<td>End To End</td>
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<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>DTW</td>
<td>Dynamic Time Warping</td>
</tr>
<tr>
<td>DBA</td>
<td>DTW Barycentre Averaging</td>
</tr>
<tr>
<td>HW</td>
<td>HardWare</td>
</tr>
<tr>
<td>LS08</td>
<td>Landsat 8 data</td>
</tr>
<tr>
<td>PCA</td>
<td>Piecewise Constant Approximation</td>
</tr>
<tr>
<td>ROI</td>
<td>Region Of Interest</td>
</tr>
<tr>
<td>S2</td>
<td>Sentinel 2 (refers to constellation)</td>
</tr>
<tr>
<td>S2-TBX</td>
<td>Sentinel 2 tool box.</td>
</tr>
<tr>
<td>SPPA</td>
<td>Sensor Performance, Product Assessment Maintenance and Operations of the Earth Observation Payload Data System</td>
</tr>
<tr>
<td>RD</td>
<td>Reference Document</td>
</tr>
<tr>
<td>TS</td>
<td>Time Series</td>
</tr>
<tr>
<td>TPX</td>
<td>Temporal Pixel</td>
</tr>
<tr>
<td>TP</td>
<td>Temporal Patchs</td>
</tr>
</tbody>
</table>

tab 2. Abbreviated Terms
4 Context and scope

The major objective of this work is the production of harmonized Sentinel 2 / Landsat 8 time series. The main purpose is therefore to create Sentinel 2 / Landsat 8 temporal block for which the pixel revisit frequency is de facto increased and from which time series might be extracted.

From functional point of view, a multi source land imaging service might be illustrated with the figure below. The figure depicts input data and processing stages that should be set up in order to produce harmonized Sentinel 2 / Landsat 8 time series.

There are several assumptions to be made regarding the input data. The input MSI/OLI data are from surface reflectance product. In addition, masks indicating water, shadow, snow, ice, cloud exist. Moreover, the data BRDF is adjusted to nadir view.

Beside input data, the description of processing such as data collection and regridding, co-registration are not within the scope of the scientific work present herein.
This document focus on the description of the algorithms involved in the consolidation of the temporal block and in a second stage, the document focus on the description of the algorithms involved in the densification of the temporal block including band pass adjustment.

The consolidation is a generic term. It refers to the recovery of values in the temporal block which are occluded at a given date. The technics for estimating missing values at a given date are various and largely promoted, they often considered the past and future observations. Nonetheless, few of them are considering the temporal structure of the time series as a key information conjointly with the information available in the spatial and spectral dimension. All the technics are pixel based; they can be assimilated as 1D (temporal) signal reconstruction.

For the purpose of this study, two categories of methods are explained and are benchmarked:

- The interpolation based methods with Linear Regression and Smoothing Filtering
- The distance based methods with the use of Dynamic Time Wrapping distance.

The DTW distance allows measure similarities between time series of a same temporal block. A time series contaminated with missing values will be compared with other time series in order to find the closest one under cost function constraint.

Beside DTW that measure similarities / alignment between time series, it is possible to associate descriptor summarizing the shape of alignment. The part dedicated to DTW discussed descriptors as well.

The densification is also a generic term. Its purpose is to populate existing consolidated time series by using data from other sensors, in particular Landsat 8 data when the existing time series are from Sentinel 2.

The baseline concept is that the Landsat 8 data should improve the analysis of the time series but in return the uncertainty of using this data should be indicated as part of the time series. It is also true for consolidation process.

At the end, the densify time series includes different category of points:

- Native points
- Estimate points by using gap filling (based on mono sensor time series, multi TBD)
- Proxy data points obtained with sensor other than Sentinel 2 but aligned to Sentinel 2 radiometric scale and also geometry.
5 Time Series Consolidation

5.1 Introduction

A temporal block is generally never fit for the purposes of time series analysis / trajectory analysis. The most common issues found in a remote sensing image is the shadow, the atmospheric contamination, the cloud and in particular cloud edge. A preprocessing is therefore needed to detect these issues in the image. Herein, it is assumed that the temporal block comes together with a temporal mask indicating for each date the valid and non-valid pixel values.

The consolidation is a process that produces a temporal block free from ‘non valid’ pixel values. From any locations in the image space of this temporal block, one consolidated time series can be retrieved for subsequent analysis.

The scope of this paragraph is therefore to describe some algorithms that are used for consolidation purposes and more specifically for interpolating missing values at a given date.

With the assumption that the input sequence (temporal block) is chronologically ordered, the following parameters might be used to estimate missing information at a given date:

- The temporal structure of the sequence,
- The spatial information,
- The spectral information.

For consolidation purposes, a majority of available methods uses neither spatial information nor the totality spectral information. Methods act locally by using relationship between past and future measurements to estimate missing value at a given date. In this category, the two following methods are described herein.

- The linear Regression,
- The smoothing filtering (Savitsky-Golay methods).

A completely different approach for consolidation of time series, is to assume that, in the temporal block, at the date and pixel location for which value are not valid, it exists valid time series. The approach is based on a similarity measurement between two time series. It is then expected that among the overall time series candidates, the time series that is closest to the input time series under similarity measurement, can be kept and its value at a given date used to consolidate the input time series.

The classic similarity measurement is the Euclidean distance. It might have been suitable for our needs; the following limitations have forced us to consider alternative methods:

- Even if non equally spaced, the temporal distribution of the two time series should be the same,
- The distance does not account for temporal variations, acceleration / deceleration

The first point is a minor limitation because in the context of consolidation, the two input times series are always from the same temporal block. The second point is a major limitation
because, within a same temporal block, it expected to find similar time series that is robust to the temporal dynamic of the time series.

For instance, the temporal dynamic of a given cover type differs depending on its geographic location over the earth, and more specifically in the scene. The reasons are various; it may solely due to the sun illumination, terrain properties. In the context of consolidation, in this typical case, the Euclidean distance outputs high distance value, it is not what is expected.

For our purposes, to select as the similarity measure the Dynamic Time Warping distance looks to be promising and it is the core elements of the proposed time series consolidation algorithms.

Comparing to time series distance (red curve above and blue curve below), the black line identify matching pair:

The left figure shows behavior of Euclidian distance
The left figure shows behavior of the DTW distance. DTW is able to re align time series.

Figure 2 -Comparison between Euclidian and DTW distance.

The algorithms does not support the processing of multi resolution time series.

Performance
Algorithms Complexity

The next part of this section is organized as follow:

- Time series “Mathematical terms and definitions”
- Time series Interpolation based methods description
- Time series distance based methods – DTW description

The mathematical terms and definition part defines a common symbol framework, in order to ease the description of the Interpolation and distance based methods given in the following.

The testing of the methods and validation are mainly reported in testing and validation document.

5.2 Mathematical terms and definitions
The purpose herein is to propose a common set of terms, in order to describe methods.

The time series is defined in first as a time-sequence of dates \((T_1, ..., T_k, ..., T_m)\) where \(m\) is the total number of dates and \(i \in [1,m]\). The temporal sampling is not regular. The dates are chronologically ordered.

Aligned to this time-sequence, a multi-dimensional sequence \(m \times n\) is defined where \(n\) is the number of observed values at a given date, \(T_k\). Hence, at a given date, the observed values are noted:

\[ B_j \text{ where } j \in [1,n] \]

Practically, these values are actually measurements extracted from the Earth Observation products originated from a specific sensor. The number of observations at a given date depends on the number of spectral bands data available in the product.

In case of Dynamic time warping methods, the number of bands to account for is an important parameter. The interpolation based methods discussed herein are not multi-dimensional and therefore they do not consider several measurements for a given date.

For our purposes, the temporal measurement (at a given date) is extracted from temporal patch (see definition above) at a given location \((x_0, y_0)\) in the image space, corresponding to pixel center coordinates. This location in the image space does not vary whatever the time. This measurement is noted more generically ‘temporal pixel’, \(TP_{x_{ij}}\), where:

- \(i\) is the index of the observation date with \(i \in [1,m]\).
- \(j\) is the index of the observed values with \(j \in [1,n]\).

The proposed methods aim at estimating an unknown values of temporal pixel, at a given space location in the temporal block. The estimated value of \(TP_{x_{ij}}\) is noted \(\hat{TP}_{x_{ij}}\).

As discussed in the introduction, the temporal mask \((Mask)\), indicating for each date the valid and non-valid pixel values, is expressed as follow in case of image mask at date \(i\) :

\[ Mask (T_i) \text{ with } i \in [1,m]. \]

5.3 Interpolation based methods description

Herein, two interpolation based methods are described; the linear Regression and the smoothing filtering (Savitsky-Golay methods).
5.3.1 Linear regression

The linear interpolation is a method extending data by fitting linear polynomials onto the actual available data. The linear extrapolation is the extension of the linear interpolation outside the range of the data.

The mask series determines where the time series data must be rebuild. This mask is then used to filter the input temporal pixel, to keep only the valid data. The linear interpolation is not a multi-band process, so the input data must also be de-interlaced before interpolation.

To rebuild the non-valid data, indicated in Mask(T_i), at the date i, the two closest points (TP_{i,j}^1, TP_{i,j}^0) that match the following criteria are selected, whatever the band, j:

\[ T_{i0} < T_i \leq T_{i1} \quad (1) \]

\[ \text{Mask}(T_{i0}) \text{ and } \text{Mask}(T_{i1}) \text{ are valid} \quad (2) \]

Then the linear interpolation is applied according the following formula:

\[ TP_{i,j} = TP_{i,j}^0 + (T_i - T_{i0}) \cdot \frac{(TP_{i,j}^1 - TP_{i,j}^0)}{(T_{i1} - T_{i0})} \quad (3) \]

In case, there is no \( T_{i0} \) or \( T_{i1} \) satisfying the two conditions (1) and (2), extrapolation using the two closest dates that match the second condition is used. The same formula is applied.

The figure below shows interpolation results when using linear regression. The original data are depicted with blue circle, the estimated data are depicted with red circle. The results of interpolation are strongly sensitive to the number of valid dates.

![Figure 3 Linear regression in a time series.](image)
5.3.2 Smoothing filtering

Savitsky-Golay is a smoothing filter, it generally use a sliding window of size 5 to try fitting a second or third order polynomial onto a noisy signal. The general algorithm principles is detailed in [RD-5].

For our experiment, the original purpose of the SG algorithm has been changed. Actually, the non-valid data are considered as noise. The estimate values are therefore the results of SG smoothing filtering process.

For that purpose we stick with the size 5 sliding window and used the two lower values and the two upper values flanking the data to rebuild. (i.e for the same band j we use the data within interval from \([T_{i-2}, T_{i+2}]\)). In this method, whereas in the general case all the value of the series should be smooth, we apply the filter only on the noisy values, the other ones will remain unchanged.

Then we proceed to a weighted least square regression over the five values using a second order polynomial regression model:

\[
y_i = a_0 + a_1 x_i + a_2 x_i^2 + \varepsilon_i (i \in [1,5])
\]  

(4)

The un-masked data is weighted with 1 whereas the masked one is weighted with 1/1E12 as we cannot set their value to zero for computation constraints.

This means that the \(TPx_{i,j}\) value is used as it is if available or set to a proxy value for the regression.

After the regression, the \(a_0, a_1\) and \(a_2\) coefficients are used to compute the value at \(TPx_{i,j}\) according to equation (5)

\[
TPx_{i,j} = a_0 + a_1 T_i + a_2 T_i^2
\]  

(5)

![Figure 4 Savitsky-Golay filter on noisy signal](image)

Notes:
- In this implementation, if other values are masked in the sliding windows of size (outside the center that must be rebuild), they will not be discarded for the computation. As a consequence, this method won’t be efficient if consecutives dates of masked data occurs.
• The uses of the size 5 sliding windows wipe away the possibility of rebuilding the two lower and the two higher dates of the timeserie. But the quality of the effective gap filling should be much better than with the linear interpolation approach.
5.4 Distance based methods – DTW description

5.4.1 Overall scheme

This DTW algorithm performs a recovering of any input time series starting from the information provided in the input temporal block.

![Figure 5 Global scheme](image)

**Input / Output data**

A temporal block and its associated temporal mask are coming together as inputs of the DTW algorithms.

The temporal mask indicated for each date, the location of values to be recovered. The accuracy of the temporal mask has an impact on the overall accuracy. Including wrong/corrupted values in a reference temporal series will definitively add noise into the consolidated temporal block. This assertions stress on the importance of having a correct preprocessing in order to detect non-valid values, so called values to be rebuilt in our context.

Regarding the output temporal block, values located at the position defined in the temporal mask has been recovered.
Collection of temporal pixels and Neighborhood temporal pixels

The temporal pixels to be recovered are extracted by using the temporal mask. Besides, the candidates are selected within the neighborhood of the temporal pixel to be recovered. The selection method is discussed below.

It is important to note that the neighborhood is of different types; it is defined as:

- The entire image,
- A specific class,
- An image region results of segmentation,
- Some pixels with a window around the temporal pixel.

Unlike the segmentation, or window, the two first types of neighborhood designed pixels with no connectivity with pixel to be estimated.

Consequently, It means that more candidates will be selected and due to different locations in the scene, the use of DTW is fully justified and the accuracy will be increased. Conversely, the complexity is increased and therefore processing time a drawback.

DTW Alignment and distance computation

The distance between time series to be rebuilt and candidate is computed for all candidates. This distance is based on similarity measures discussed below.

‘N-DTW’ Metrics and descriptors

The DTW metrics contains two values: the warping path / alignments and the distance associated to the path. These two metrics are described in part dedicated to the similarity measure description. Other descriptors might be used in order to exploit the shape of the optimal warping path.

Minimization

The minimization step relies on the DTW metrics in order to select one temporal pixel among all available candidates.

Aggregation

The aggregation is the last processing stage. It consists in injecting the data of the selected candidate into the temporal pixel to be recovered at the expected dates.
5.4.2 Collection of temporal pixels and Neighborhood

This approach consists in using the DTW metrics in order to find similar temporal pixel within the rest of the time series. This pixel will then be used as support for the current pixel rebuilding.

All the pixels requiring to be recovered are collected among the full time series and they will be process independently. For that purpose we also collect the temporal pixel candidates to support the aggregation step.

The temporal pixel candidates are selected from the neighborhood of the pixel to recovered. This neighborhood can be extended to the whole time series imprint or retrained to a specific labeled region. For computation performance reason we might prefer to provide a previously existing regions image.

We can define the neighborhood \( N \) of the temporal pixel at the position \( (x_0, y_0) \), as \( \{ (x_k, y_l) \in N \mid x_k \neq x_0, y_l \neq y_0 \} \)

This step of the process includes a filtering step as current pixel or the neighborhood pixels might be occluded for one or several date. For each collected temporal pixels both to be recovered and candidates, the temporal mask is apply to keep only the valid data on which the DTW will be computed.

5.4.3 The Similarity measure description

The dynamic time wrapping or DTW is a distance measure between two time series. Unlike the standard Euclidian distance, it consists in re-aligning the two series that might have temporal distortion.

The DTW is used to align sequence on a common time-axis by using dynamic programming. The optimal warping path (between two series) satisfies three basic constraints: boundary condition, continuity and monotonicity ([RD-2]). The optimal warping path is found as satisfying the minimal overall distance.

A warping path is formally described as a two-component vector including for each index: the input temporal pixel date and its corresponding date in the candidate sequence.

The size of both sequences being the same, at the index \( k \), \( k \in [1, l] \) where \( l \geq m \) and \( l \leq 2m \), the value of the warping path is \( (i, i') \), with both index \( i \in [1, m] \). The length of the warping path is in the range of \( m, 2m \), where:

- A length of \( m \) indicates a perfect alignment and no distortion, i.e. the warping path corresponds to the diagonal,
- A length of \( 2m \) represents the maximum acceptable taking into account continuity, boundary constraints.

The figure below shows how the warping between to two time series \((Q, C)\) is represented (right). Geometric distortions between both series exist. The warping reflects these differences.
Time Series Consolidation and Densification using dynamic time warping technic

5. Time Series Consolidation

Figure 6 illustration of the warping path between two series C et Q

This overall distance of a given warping path is very similar as the Euclidian distance, except that the indexes of series are adjusted according to the indexes of the resulting warping path.

The individual distance at a given index k of the warping path (where the index i is associated to the index i’) is computed by using the multi-spectral information with the following formula:

$$dist_k = dist(i,i') = \sqrt{\sum_{j=1}^{m}(C_{ij} - Q_{i'j})^2}$$

The overall DTW distance is computed as the sum of the individual distance computed at each point of the warping path:

$$DTW = \sum_{k=1}^{l} dist_k$$

Note that beside DTW distance and warping path, additional information can be computed, as discussed in §5.4.5.

The DTW considers temporal dimension as part of the distance and is particularly convenient for the analysis of the time series. In earth observation, despite the fact that each studied product has a timestamp, a same phonological event might occur on a distorted period. For example, a forest observation on the same period may vary according the geographical location, the temperature variation or even the observation angle. The DTW will help finding a good matching between temporal pixels.

Another interesting property is that the DTW is also compatible for computing distance between two series of different length. In our case, the two sequences; temporal pixel to be
recovered and neighborhood temporal pixels do not have the same length. Indeed, dates with no valid measurement are removed. It is a reason for which the DTW is justified.

A significant overlapping (90%) should exist between temporal periods of the two sequences.

In case of two sequences having a different temporal density, the algorithm perform well, within the condition that within each sub period of the densely populated time series (S2), there is a corresponding value in the sparsely populated time series (LS08).

The DTW is computed for all the candidates. The best candidate is then selected by using minimization approach, as described below.

### 5.4.4 Minimization

Once the DTW metrics are computed between the temporal pixel to recover and all the collected temporal pixels candidates, the pixels responding to the following criterions are selected:

\[
DTW(TP_x(x_0, y_0), TP_x(x_k, y_l)) = \min_{(x_k, y_l)} DTW(TP_x(x_0, y_0), TP_x(x_k, y_l))
\]

(6)

\[
Mask(T_i) \text{ are valid}
\]

(7)

Among these pixels we select the best candidate as the one fulfilling both the (6) and (7) conditions.

Instead of keeping only one value, the one from the minimum, it is also possible to keep a “short list” including the best candidates. There are at least two benefits:

- To analyze derivative up to candidate - in particular as confidence index
- To select other one, if the best one reveals to be finally corrupted because temporal pixel selected is itself corrupted ...

It is important to note that the current selection method takes only into account the DTW distance between the pixels. The temporal distortion apply onto the pixels to obtain the distance is not exploited. Adding a constraint on the warping path during the selection amounts to have an a priori on how important is the effective temporal distortion within the data. This is the purpose of the descriptors describe in the following section can be use.

### 5.4.5 The Descriptors

The descriptors are useful information to characterize alignment between two time series. There are three main descriptors to consider:

- The warping path length. Its indicates how many temporal distortions there is along the path
- The integral of the warping path regarding the diagonal axis. It indicates the global effort to align the two series, where the distance metrics indicates the sum of local efforts.
• The derivative of the warping path. Like the integral descriptors the derivative, indicates a global effort for the temporal distortions. But as its local value depends on the previous state, it indicates also the variation within the global effort and the direction of the effort.

In our context, these descriptors could be used for the final selection as constraint for the minimization. The constraints will influence directly the performances of the method. Note that the current method does not implement descriptors.

5.4.6 Simulation

Below we demonstrate the selection method for the reconstruction of a pixel having a single occulted date. We rely on the Corin Land Cover (CLC) in order to compute the DTW metrics only on the pixels belonging to the same class. Within the illustration we consider only the NIR band for the temporal pixels values.

![Figure 7 Exemple of a temporal pixel to rebuild](image)
5. Time Series Consolidation

5.4.7 Computational Performance

**DTW Parameters**

The complexity of the algorithm is \( M \times N \) for the number of warping path, where \( M \) and \( N \) are the length of the two input series.
This complexity can be reducing by applying a constraint on the search window for the warping path. For example we can define the range of the window by using $d_{\text{max}}$, the max distance to the diagonal for aligning onto the series of size $N$. The complexity is then reduce to $(M \times 2 \times d_{\text{max}}$).

The accuracy of the DTW depends on the temporal density of the two series. Their profiles shouldn’t contain temporal gaps affecting their global shape. For sure depending also on the cover types to be rebuild.

Moreover, the accuracy of the DTW depends on the noise level within the profile, reason for which the pre-processing of the temporal block (SR measurements, with nadir brdf adjustment ...) is more than expected for an accurate determination of the mask.

**Time processing performances:**

The systematic computation of the DTW with all other available pixels maybe quite time consuming regarding the DTW complexity and depending on the length of the time series.

The solution in order to decrease the processing time is to reduce the neighborhood of the pixels. This reduction can be achieved by computing segmentation or classification of the temporal series content. We have listed three types of approaches in the table below and compared their potential. These methods reduce the neighborhood of a pixel to the pixel belonging to the same segment or class.

For optimization, i.e reduced num, DBA and Kmeans segmentation described in [RD-6] could be used but the accuracy may become reduce and the gain in the overall time processing is not guarantee.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Method</th>
<th>Comments on accuracy</th>
<th>PRO</th>
<th>CONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Pre- segmentation on single product</td>
<td>K-mean or meanshift</td>
<td>Reduce drastically the processing time.</td>
<td>Improve processing time for low implementation and complexity cost</td>
<td>No guarantee on the temporal stability of the segmentation (computed on one frame)</td>
</tr>
<tr>
<td>Classification map</td>
<td>CLC classification</td>
<td>Reduce drastically the processing time.</td>
<td>Improve processing time</td>
<td>Depends on the classification accuracy and the classes homogeneity</td>
</tr>
<tr>
<td>K-means + Pre-</td>
<td>Reduction ratio</td>
<td>Improve</td>
<td>This</td>
<td></td>
</tr>
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</table>

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<table>
<thead>
<tr>
<th>dba</th>
<th>classification using K-means</th>
<th>SIZE_OF_EACH_CLASSES/SIZE_OF_THE_IMAGE</th>
<th>processing time</th>
<th>classification can be as time consuming as the DTW itself. (same complexity)</th>
</tr>
</thead>
</table>

**Figure 10 Solutions for reducing the number of candidates**

**Method accuracy:**

- The accuracy of this approach depends on the capacity of the DTW to find within the time series another temporal pixel with same (or at least similar) temporal profile. And this capacity is directly linked to the content of the temporal frames.
- We can expect that the multi-band aspect of this method improve its accuracy compares to the two previous ones.
6 Time Series Densification Methods

To be continued in [PHASE 2] – WP300

- End of the document -