**Nonlinear diffusion filtering of the GOCE-based satellite-only MDT**

**Abstract**

We present nonlinear diffusion filtering of the satellite-only mean dynamic topography (MDT) obtained as a combination of the GOCE-based geoid models and mean sea level models provided by satellite altimetry. Our numerical approach is based on filtering on a closed surface using linear and nonlinear diffusion equations. We define a surface finite volume method (SFVM) to approximate numerically parabolic partial differential equations on closed surfaces, namely on the Earth’s surface. Such computational domains are approximated by polyhedral surfaces created by planar triangles and we construct dual co-volume grids. On the co-volumes we define a weak formulation of the problem by applying Green’s theorem to the Laplace-Beltrami operator. Then SFVM is applied to discretize the weak formulation, where we consider a piece-wise linear approximation of a solution in space and the backward in time discretization. Later on, we extend the linear surface diffusion to the regularized surface Perona-Malik model, which represents the nonlinear diffusion equation.

In our numerical experiments we focus on reducing the stripping noise from topography (MDT) obtained as a combination of the GOCE-based geoid models while the noise is effectively reduced.

**Linear diffusion filtering on a closed surface**

- Linear filtering of data: usually convolution with the Gauss function
- Gauss function is also a fundamental solution of the linear heat (diffusion) equation: to replace the convolution of filtering by solution of the heat equation

\[ c_{i, j} = \frac{1}{N} \sum_{k=1}^{N} c_{k} \]

- parabolic PDE

**Numerical discretization – surface finite volume method**

Weak formulation: by applying Green's theorem

\[ \frac{\partial}{\partial t} \int_{\Gamma} u \, ds = \int_{\Gamma} \nabla u \cdot \mathbf{n} \, ds + \int_{\Omega} \nabla \cdot \mathbf{q} \, dx \]

Discretization in time: the backward Euler time difference

\[ \frac{u_{n+1} - u_{n}}{\Delta t} = \nabla \cdot \mathbf{q}_{n+1} \]

Discretization in space: triangulation of the closed surface + dual co-volume grid

Triangulation of a sphere: triangulation of the closed surface + dual co-volume grid

Linear representation of the solution \( u \) on a triangle

- constant surface gradients

\[ u_{i} = \frac{1}{2} \left( u_{i+1} + u_{i-1} \right) \]

Implict numerical volume for the linear surface diffusion

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Regularized surface Perona-Malik model: nonlinear PDE

\[ \frac{\partial u}{\partial t} = \nabla \cdot \left( \frac{1}{\phi} \nabla u \right) \]

where

\[ \phi = \left( u + u_{0} \right)^{2} \]

Edge detector

\[ c(x, y, t) = \frac{1}{\pi \Delta x \Delta y} \int_{-\Delta y}^{\Delta y} \int_{-\Delta x}^{\Delta x} e^{-\frac{(x-x_{0})^{2}+(y-y_{0})^{2}}{2 \sigma^{2}}} \, dx \, dy \]

Weak formulation: by applying Green's theorem

\[ \frac{\partial}{\partial t} \int_{\Gamma} u \, ds = \int_{\Gamma} \nabla u \cdot \mathbf{n} \, ds + \int_{\Omega} \nabla \cdot \mathbf{q} \, dx \]

Semi-implicit discretization scheme for the nonlinear surface model

\[ \frac{u_{n+1} - u_{n}}{\Delta t} = \nabla \cdot \left( \frac{1}{\phi_{n+1}} \nabla u_{n+1} \right) \]

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**References:**


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